MID-GRIP FORCES AND MOMENTS ESTIMATED FROM POLE DEFORMATION IN POLE VAULTING

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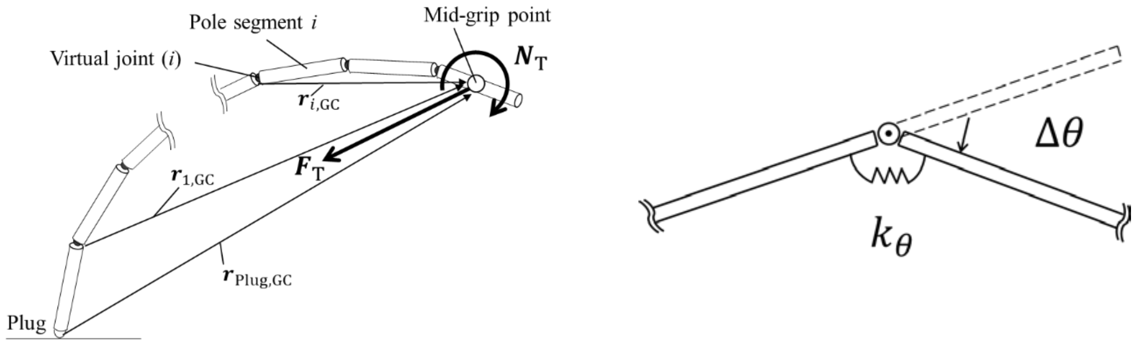
The pupose of this study was to propose a method which estimates the resultant force and moment from the shape of a deformed pole based on a quasi-static equilibrium equation. The force and moment were assumed to be exerted, at the middle point of both hands, on the pole. The pole was modelled as a series of 11 rigid segments connected to the adjacent segment via a virtual rotational joint with a rotational spring component. The stiffness of the spring was identified from a static load test. The resultant mid-grip force and moment were calculated from a static force-moment equilibrium equation. The results indicate that the kinetic values obtained by the proposed method show similar values reported in a previous study which utilized a force plate.

KEYWORDS: resultant force, coupling force, pole stiffness, closed loop problem

INTRODUCTION: The pole-vault is a track-and-field event in which an athlete achieves height largely from energy stored in the vaulting pole. Vaulters bend the pole as much as possible to utilize a large restoring force which raises the vaulters' body up in the last half of the vaulting motion. The pole bending is achieved by the resultant force and moment (i.e. coupling force) on the grip-handle part of the pole by both hands of the vaulters. Thus, knowledge of the kinetic variables as well as the kinematics (Angulo-Kinzler et al., 1994) and energetics (Arampatzis et al., 2004; Schade et al., 2006) provide useful information for vaulters to investigate their vaulting technique. Several studies on the kinetics of pole vaulting have been reported, in which 2-D exerting forces and moments by the upper-side hand (McGinnis, 1986) and 3-D resultant forces and moments exerted by both hands (Morlier and Mesnard, 2007) were investigated. Since these studies used a force platform and conducted an inverse dynamics calculation for a whole-body model, it would be difficult to implement the methods without the force platform or whole-body kinematics data for obtaining the kinetic information of both hands. In contrast, the shape of the deformed pole should be able to provide the resultant force and moment exerted by both hands due to the elasticity of the pole. Therefore, the purpose of this study was to propose a method in which the resultant force and moment exerted on the grip center of the hands was estimated from the pole's deformation.

METHODS: Three male pole vaulters (Mean ± SD: 1.74 ± 0.05 m, 65.3 ± 8.6 kg, Personal Record (PR): 4.93 ± 0.35m, pole length: 4.60 m for participant 1 and 2 and 4.90 m for participant 3), who were members of a university athletic team, participated in this study. They performed 5-8 trials at 95-98% of their PR. One trial with the highest subjective rating per participant was analyzed. Kinematic data (47 markers on the body; 20 and 26 markers on the poles of 4.6-m and 4.9-m length, respectively) were captured with a motion capture system (Vicon-MX; 20-Camera; 250 Hz). The pole bending phase was defined as the period from the athlete's takeoff to the instant of the maximum bending of the pole.
The pole was modelled as a series of 11 rigid segments, in which each adjacent segment was connected via a rotational virtual joint with a rotational spring component (Figures 1a and b). The stiffness of the rotational springs, whose values were assumed to be consistent over the pole, were identified from a static-load test of the pole. The forces and moments exerted by both hands were assumed to be applied to the middle grip of the pole, which was the centre point of both hands.



(a). A planar pole deformation model **(b). A virtual joint with a rotational spring**

Figure 1: Schematic representations of a deforming pole model

Since the lower-end of the pole (i.e. plug in Figure. 1a) can only exert a force on the ground, the deformation of the pole was assumed to be constraint on the bending pole plane. Under the above assumption, the virtual joint can only rotate about the axis normal to the bending pole surface. The joint torque vector $\boldsymbol{\tau}$, consisting of the individual joint torques $\boldsymbol{\tau} = [\tau_1, \tau_2, \cdots, \tau_{nj}]$, was calculated from the product of the angular displacement vector $\Delta\boldsymbol{\theta} = [\Delta\theta_1, \Delta\theta_2, \cdots, \Delta\theta_{nj}]$ and the matrix of rotational spring stiffness at the individual joints $\boldsymbol{K_\theta} = diag\ \{k_\theta, k_\theta, \cdots, k_\theta\ \}$ as:

$$\boldsymbol{\tau} = \boldsymbol{K_\theta}\Delta\boldsymbol{\theta} \tag{1}$$

The static equilibrium equation, being defined on the bending pole surface, between resultant force vector $\boldsymbol{F}_\mathrm{T}$ and moment $\boldsymbol{N}_\mathrm{T}$ at mid-grip point and the joint torque vector $\boldsymbol{\tau}$ can be written in the following form as:

$$\boldsymbol{\tau} = \boldsymbol{J}^\mathrm{T}\boldsymbol{F}_\mathrm{T} + \begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{E}\\ \vdots\\ \boldsymbol{E}\end{bmatrix}\boldsymbol{N}_\mathrm{T} \tag{2}$$

where $\boldsymbol{J}$ is the Jacobi matrix consisting of the outer product matrices of position vectors pointing to the middle grip from the virtual joint centres. As no moment acts on the plug, an equilibrium equation with respect to $\boldsymbol{F}_\mathrm{T}$ and $\boldsymbol{N}_\mathrm{T}$ can be expressed as:

$$\boldsymbol{r}_{\mathrm{plug,GC}} \times \boldsymbol{F}_\mathrm{T} + \boldsymbol{N}_\mathrm{T} = 0 \tag{3}$$

where $\boldsymbol{r}_{\mathrm{plug,GC}}$ is the position vector pointing to the middle grip from the plug. Combining eqs. (1), (2) and (3), one can derive an equation solving the resultant force vector $\boldsymbol{F}_\mathrm{T}$ with use of a pseudo inverse matrix as:

$$\boldsymbol{F}_\mathrm{T} = (\boldsymbol{B}^\mathrm{T}\boldsymbol{B})^{-1}\boldsymbol{B}^\mathrm{T}\boldsymbol{K}_\theta\Delta\boldsymbol{\theta} \tag{4}$$

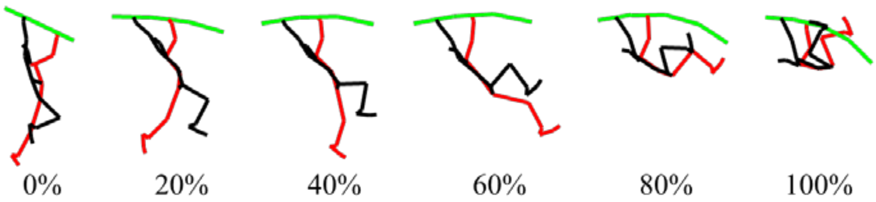
$$\boldsymbol{B} = [0 \quad 0 \quad 1]\begin{bmatrix}[\boldsymbol{r}_{1,\mathrm{GC}}\times]\\ \vdots\\ [\boldsymbol{r}_{i,\mathrm{GC}}\times]\\ \vdots\\ [\boldsymbol{r}_{nj,\mathrm{GC}}\times]\end{bmatrix} - \begin{bmatrix}1\\ \vdots\\ 1\\ \vdots\\ 1\end{bmatrix}[0 \quad 0 \quad 1][\boldsymbol{r}_{\mathrm{plug,GC}}\times] \tag{5}$$

Since the vector $\boldsymbol{F}_\mathrm{T}$ was constrained on the bending pole plane, the rank of the pseudo matrix $\boldsymbol{B}$ was 2. Thus, the inverse matrix of the matrix $\boldsymbol{B}$ was calculated using a singular value decomposition. Then, the vector $\boldsymbol{N}_\mathrm{T}$ was calculated from the $\boldsymbol{F}_\mathrm{T}$ by using eq.(3).

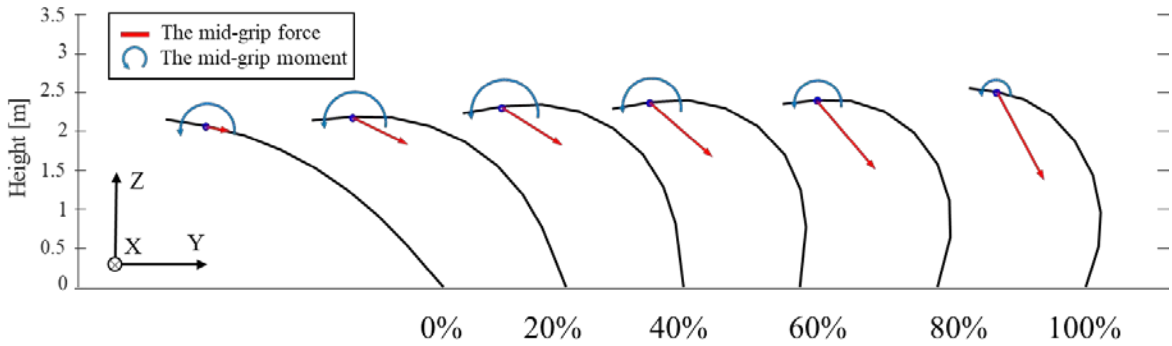
RESULTS AND DISCUSSION: Figure 2 shows schematic representations of the estimated mid-grip force and moment exerted by Vaulter 2. The length of the linear arrows and radius of the round arrows denote the magnitudes of the mid-grip force and moment vectors, respectively. The direction of the mid-grip force changed from forward to downward. The mid-grip moment was exerted about the normal axis of the Y-Z plane from just after takeoff (0% normalized time). The mid-grip moment about the normal axis of the X-Z plane increased as the pole rotated in the counter-clockwise direction.

Figure 3 shows the force-time history curves of the vertical component of the mid-grip force vector for all participants. Every vaulter exerted a downward force on the ground, whose maximum values were between -660 N and -765 N at the instant of the maximum bending of the pole.

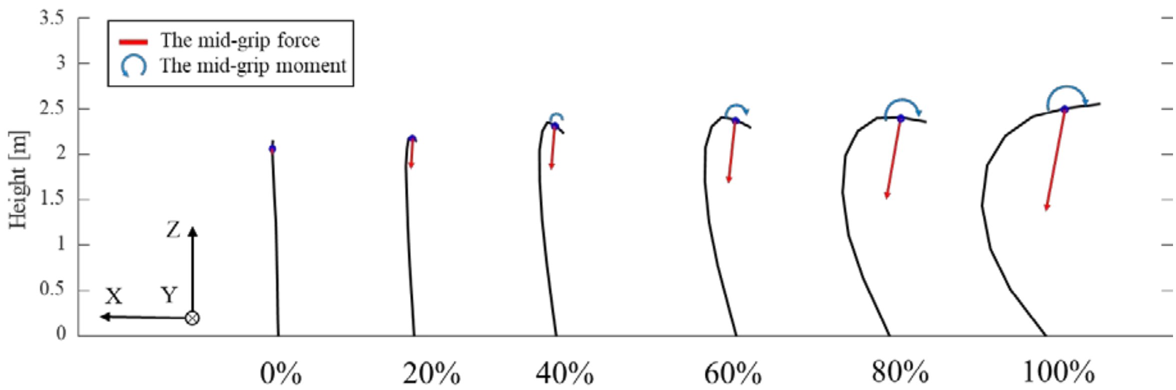
Figure 4 shows the moment-time history curves of the estimated moments in the pole bending plane. The mid-grip moment values at 0% normalized time were between -230 Nm and -385 Nm, and the maximum values in magnitude were between -384 Nm and -490 Nm.



(a). Stick figure representation of pole vaulting



(b). Stick figures of the estimated mid-grip forces and moments on the Y-Z plane



(c). Stick figures of the estimated mid-grip forces and moments on the X-Z plane

Figure 2: Schematic representations of the estimated mid-grip forces and moments (Vaulter 2), where X-, Y- and Z-axis respectively denote transverse, horizontal approach and vertical directions.

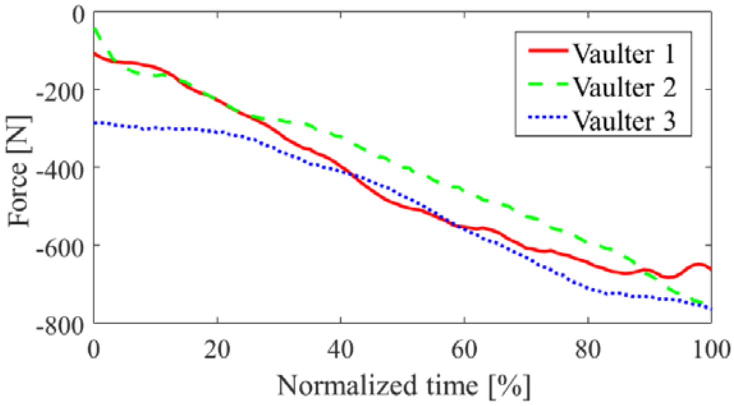


Figure 3: Force-time curves of the vertical component of the estimated mid-grip force vector.

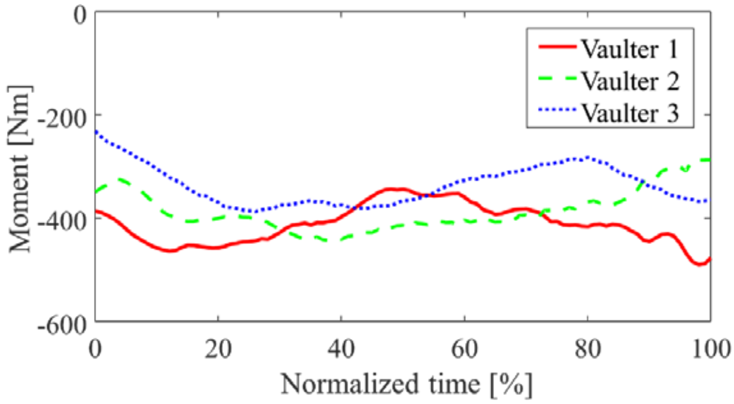


Figure 4: Moment-time curves of the estimated mid-grip moment exerted in the pole bending plane.

The ground reaction forces obtained from the estimated mid-grip forces were similar to those reported by Morlier and Mesnard (2007), which utilized a whole-body inverse dynamics approach, around the instant of the maximum pole bending. The reported range of ground reaction forces were between 600 and 850 N around the instant. Both hands and a pole create a mechanical closed loop system. One cannot decompose the mid-grip forces or moments exerted along the pole into individual forces and moments exerted for each hand. Hence, reported forces and moments for the mid-grip position do not reflect the stress at each hand.

CONCLUSION: A method for estimating the resultant mid-grip forces and moments from the shape of the deformed pole was presented. This method can be conducted without a force platform and inverse dynamics calculations for the whole body. However, the estimated forces and moments should be verified by using a force platform or force sensors in future studies.

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