CROSS RECURRENCE QUANTIFICATION ANALYSIS OF INTER-LEG RELATIONS ACROSS THE GAIT TRANSITION

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This study quantified characteristics of the deterministic properties of inter-leg movements over the gait transition. The purpose was to further understanding of the nonlinear characteristics of the gait of a healthy individual. A participant locomoted on a treadmill as the speed steadily increased from 0 to 18 km/h over a 120 s period. Position of the approximate center of rotation of the bilateral toe, ankle, knee and hip, were collected. The mathematical addition of the four markers representing the “free leg” in the direction of travel (x direction) provided “left leg” and “right leg” variables. Each leg was embedded with a dimension of 3 and time lag of 18 points to create the Taken’s vector. Cross Recurrent Quantification Analysis (RQA) was then performed. Two distinct phases in the combined dynamics of the leg-leg system were observed, with lower determinism in the walking phase compared to running, separated by a clear, sharp transition. Results indicate that determinism in the cross dynamics of the two legs seems to play a role similar to an order parameter for the walk-run phase transition. This work increases our understanding of the nonlinear dynamics characteristics of the gait transition.

KEYWORDS: non-linear dynamics, walking, running, Taken’s Theorem

INTRODUCTION: Our understanding of human gait has been predominantly developed through biomechanical analyses using linear mathematics (Van Emmerik et al, 2016). While these methods facilitate an understanding of the mechanical characteristics and constraints that are produced and satisfied during gait, they do not describe the nonlinear characteristics of our biological system. Both experimental and robotics fields have identified that an epistemological shift towards understanding the dynamics of a system within constraints, where redundancy and variability of the system are used to satisfy collective dynamics is mathematically, theoretically and practically more fruitful (Iqbal et al., 2014). From a dynamical systems perspective, nonlinear dynamics methods are used to explore and understand pattern stability, transitions between states, and deterministic and stochastic processes at different spatio-temporal scales (Van Emmerik et al., 2016). The aim of this study was to quantify characteristics of the determinsitc properties of inter-leg movements over the gait transition. The purpose was to further understanding of the nonlinear characteristics of the gait of a healthy individual.

METHODS: Ethical approval was gained from the host University ethics committee. A male participant (age 28 years, mass 75.2 kg, height 1.90 m, regular runner ~45 mins x 4 times per week) locomoted on a motorized treadmill as the speed steadily increased from 0 to 18 km/h over a 120 s period. 3D position of the approximate center of rotation of the bilateral toe, ankle, heel and knee were collected (CODAmotion, Charnwood Dynamics Ltd, UK; 100 Hz). All analyses were performed in R (http://www.r-project.org). The mathematical addition of the four markers representing the “free leg” in the direction of travel (x direction) were computed after removing the offset (each marker oscillated around zero); the resulting variable was subsequently referred to as the “left leg” and “right leg” (Williams and Vicinanza, 2017). Taken’s vector of each leg was then built based with an embedding dimension of 3 and a time lag of 18 points. Embedding dimension was calculated using the false nearest neighbor algorithm and the embedding delay was determined by finding the first minimum of the average mutual information algorithm (Fraser and Swinney, 1986; Kennel, Brown and Abarbanel, 1992; Perc, 2005).
Recurrent Quantification Analysis (RQA) is a non-linear analysis method for quantifying the number and duration of recurrences of a dynamical system in its phase space trajectory

(Webber and Zbilut, 1996). In this study Cross RQA (cRQA), a bivariate extension of RQA, was used to study the phase space trajectories of the two systems (left and right leg) in the same phase space (Marwan and Kurths 2002). cRQA plots compared occurrences of similar states of two systems, highlighting and quantifying similarities of the pattern of dynamical evolution.
Three variables were calculated from the cRQA plots. In calculating all three variables, window size was set to 280 points (2.8 s) and window step to 40 points (0.4 s). Firstly, the proportion of recurrent points forming diagonal lines quantified the determinism in the combination left leg-right leg structures in the phase space, where determinism can be defined as how the trajectories will be close to each other i.e. the mean prediction time. Secondly, the average length of the line structure quantified the average length of these deterministic structures. Lastly, length of the longest diagonal line were calculated, quantifying the size of the largest fraction of the trajectory that was deterministic.

RESULTS: Figures 1a, b shows a detail of the phase-space trajectories of the left and right leg during walking (12 seconds, from t=10s to t=22s). The orbits spiraling around an attractor structure are an indication of the stable dynamics.
Figure 1c represents the cRQA plot between the left and right leg across the period of gait transition (20 s, from t=50s to t=70s). The darker square structure in the top left is related to the fast walk just before the gait transition; the lighter square structure visible in the bottom right is related to the running phase, after the transition. The sharpness of the transition and how the determinism (i.e. the number of points aligned in diagonals) increases in the running phase compared to the walking phase should be noted. In addition, the darker square section related to the end of the walking phase starts to became more structured approaching the transition, with the creation of longer and clearer diagonal structures in the fast walk.
Figures 1d, e, f show the windowed cross-determinism, mean and maximum length of the diagonal. The three graphs display how relevant non-linear dynamic parameters clearly change as the velocity increases from 0 to 18 km/h, highlighting the existence of two distinct phases in the combined dynamics of the leg-leg system, with lower determinism and shorter diagonals in the walking phase. In addition, the Figures 2d, e, f show a clear, sharp transition related to the sudden increase in determinism and length of the diagonals.
In particular, Figure 1d shows how cross-determinism, defined here as the fraction of points in diagonal line structures of length > 2, firstly decreases during the walking phase as the velocity approaches the gait transition and then suddenly increases and stabilises after the transition, during the running phase. Figure 1e displays the change in the average diagonal length, which follows an inverted U shape during the walking phase, with diagonal lengths. The gait transition is clearly visible here as well and characterised by a doubling of the average diagonal length. Finally the running phase displays two inverted U shapes.
Table 1 contains a summary of the three RQA metrics just described, analysed for a sample of 20 seconds during the walking (t=20s-40s) and running (t=80s-100s) phases.

Table 1: Cross RQA metrics for a 20s sample during walking (t=20s-40s) and running (t=80s-100s).

Cross RQA metric	Walking phase	Running Phase
Fraction of points in a diagonal line structure of length >2 (Determinism)	70.81%	94.41%
Average length of the diagonal	5.75	11.44
Maximum length of the diagonal	53.73	206.76

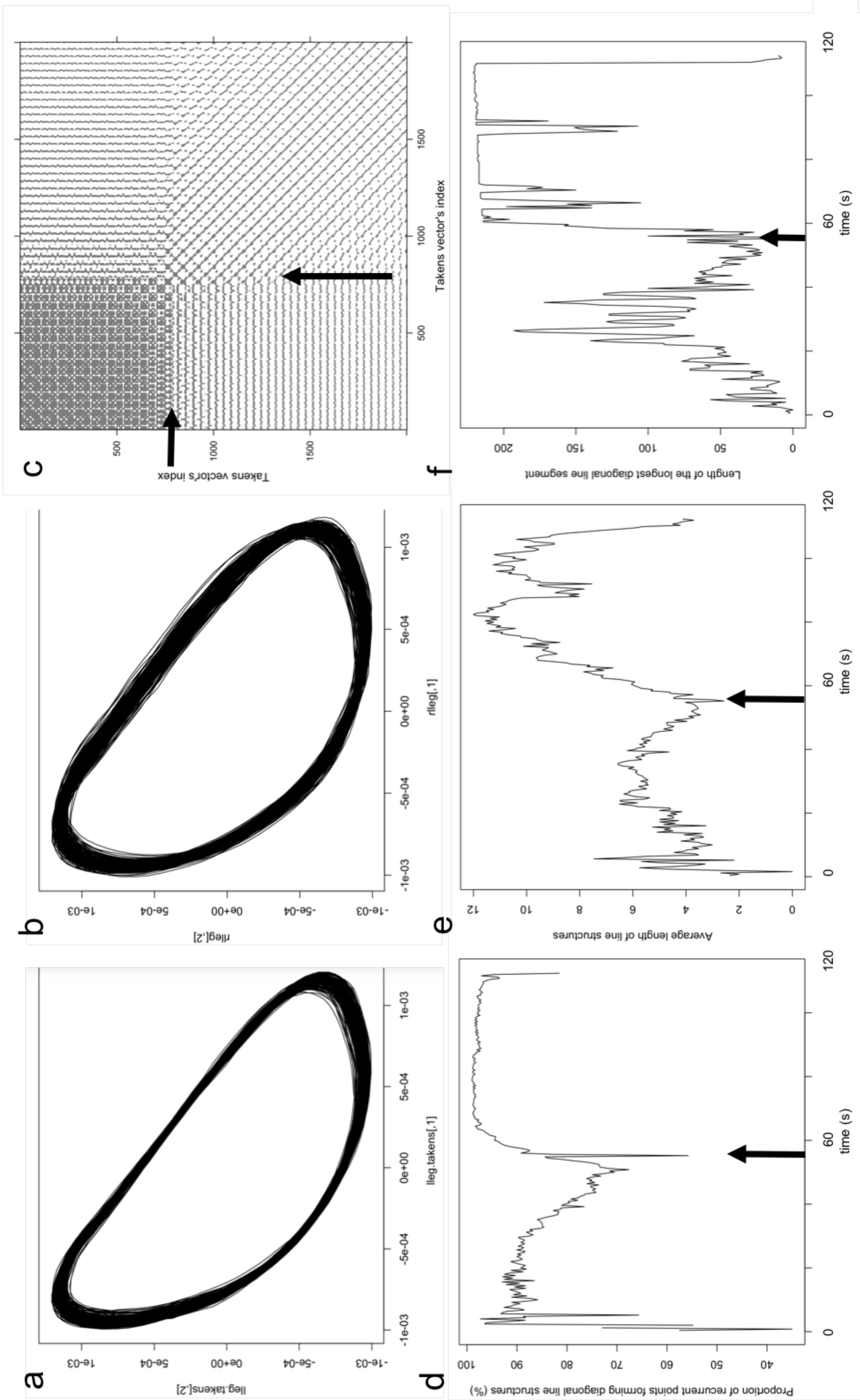

Figure 1. Phase space trajectories of the left (a) and right (b) leg during the walking phase (12 seconds, from t=10s to t=22s); cRQA plot (c)between the left and right leg across the period of gait transition (20 seconds, from t=50s to t=70s); and the proportion of points forming diagonal lines (d), average length of line structures (e), and length of the longest line structure treadmill speed increased from 0-18 km/h over 120 s.

DISCUSSION: Based on cRQA, two distinct phases in the combined dynamics of the leg-leg system were observed which corresponded to the walk and the run phase of gait across a steadily increasing speed. The walking phase was characterized by lower determinism compared to the running phase. Determinism in this case was defined as the mean prediction time which is present for the mutual evolution of the two trajectories (left leg, right leg) in the phase space. The occurrence of a cross-recurrence happens when the patterns of the limit cycle of each leg occur in the same area of the phase space. Based on the amount of this recurrence the dynamical evolution (possibly equivalent to the mechanical symmetry) can be explored. Determinism in the mutual dynamics between the legs is presented here as a theoretically grounded way to capture the collective dynamics in gait.

Our aim was to quantify characteristics of the determinsitc properties of inter-leg movements over the gait transition. While order parameters have been suggested, these are not currently based on nonliear dynamics, e.g., relations between the kinetic energy and potential energy (Segers et al., 2012). The current approach provided a way to explore the dyanmics that occured before and after the gait transition and further understanding of the nonlinear characteristics of the gait of healthy individuals. The cross recurrence plot, displayed in Figure 1c, highlights the structural differences between walking and running across the transition. The sudden and clear appearance of long diagonal structures in the running phase (bottom right square), indicates a swift increase in the occurrences of similar states in the dynamics of the two legs. This brisk change is in line with a deep transformation in the dynamical structure of the system happening across the gait transition. The Figures 2d, e, f show a clear, sharp transition related to the sudden increase in determinism and length of the diagonals. Figures 1d, e, f highlight different aspects of the non-linear dynamics of the walking and running phases. Figure 1d shows how the number of points in diagonal lines decreases when the locomotion speed increases during walking and suddenly grows at the transition (from 66.7% just before, to 95.8% just after it). Fast walking is characterized by a lower number of matching patterns (recurrences) in the phase space. On the other hand, the subsequent stable plateau (centered around 92.63% with a SD of 3.35%) in the running phase indicates a very stable, deterministic dynamical system. Figures 1e and f display a similar behaviour: both the average and maximum length of the diagonals span over a quite different range in the walking and running phase. The average number of points almost doubles during running, and the maximum length almost quadruplicates, from a mean value of 55 to 205 points, indicating a much better predictability of the combined leg-leg dynamical system (the max prediction time quadruplicates as well from 0.55s to 2.05s). The determinism plays a role similar to an order parameter for the walk-run phase transition, where the velocity is the control parameter. This result opens a discussion as to where the order parameter for walking and running might lie.

CONCLUSION: The purpose of this study was to further understanding of the nonlinear characteristics of the gait of a healthy individual. The cross RQA naturally defines two regions, one related to walking with shorter diagonals, less recurring segments, and the other to running, with much longer prediction time, increased determinism and higher average length of the diagonals. The two regions are separated by a sharp transition, characterized by a sudden increase of the first derivative (>3 X the SD), in line with the assumption that determinism behaves like an order parameter for the walk-run phase transition.

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