SUPERPOSITION PRINCIPLE APPLIED TO THE OPTIMIZATION OF KICK-TO-STROKE RATIO OF BACKSTROKE

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Backstrokers acknowledge there is an apparent rhythm in swimming that is characterised by the periodic motion of the arms and legs, which must be optimally sychnronized for effective performance. However, unlike the freestyle crawl where the optimal kick-stroke ratio is established at 3:1, there is a limited understanding of the optimal ratio for backstrokers. By separating the velocity contributions of the kick and stroke, the superposition principle can be utilised to find the optimal kick-stroke ratio to maximize swimming distance, the area below the superimposed velocity curve. This paper intends to mathematically establish this ratio and thus improve the performance of backstrokers.

KEYWORDS: impulse, stroke kick ratio, periodicity.

INTRODUCTION: Swimming is a widely acknowledged sport in all areas of the world. It has recently been adopted by the Korean Education Ministry as a compulsory subject for physical education teachers with specialist domains, such as survival swimming, along with competitive swimming, now trending in that country (Korean Ministry of Education, 2017). Alongside this increasing popularity in South Korea is the growing application of sport science to swimming with a view to significantly enhancing performance. In swimming, where medals and ranks are distinguished by the tenth of a second, statistics and biomechanics are increasingly used to help swimmers gain a competitive advantage. Therefore, it is only imperative that amateur swimmers are also provided with guidelines, evidenced by applied scientific research, to enhance their swimming performance.

METHODS: The propulsion of a swimmer is a function of the speed of the arms and legs that push through the water, noting that this water follows the limbs to create a momentum change in the swimmer as well. Therefore, through monitoring the periodic motion of the swimmer, we can infer that the positive (propulsion) velocity contribution from each limb will be periodic as also (Bilinauskaite et al., 2013; Nauwelaerts, Stamhuis & Aerts, 2005). In this paper, we approximate this velocity contribution as sinusoidal. Then, we isolate the contributions of the arms and legs, with an assumption that it is reasonable for a swimmer to change stroke rate given a fixed kick rate, and vice versa. The total velocity is expressed by the following expression:

$$v(\mathrm{t}) = |A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t + \varphi)|, \tag{1}$$

where A_1 is the maximum propulsion velocity generated by the arms of the swimmer and the A_2 is the maximum velocity generated by the kick of the swimmer, with stroke and kick rate defined as ω_1 and ω_2, respectively. The value φ is the phase difference between two sinusoidal functions.

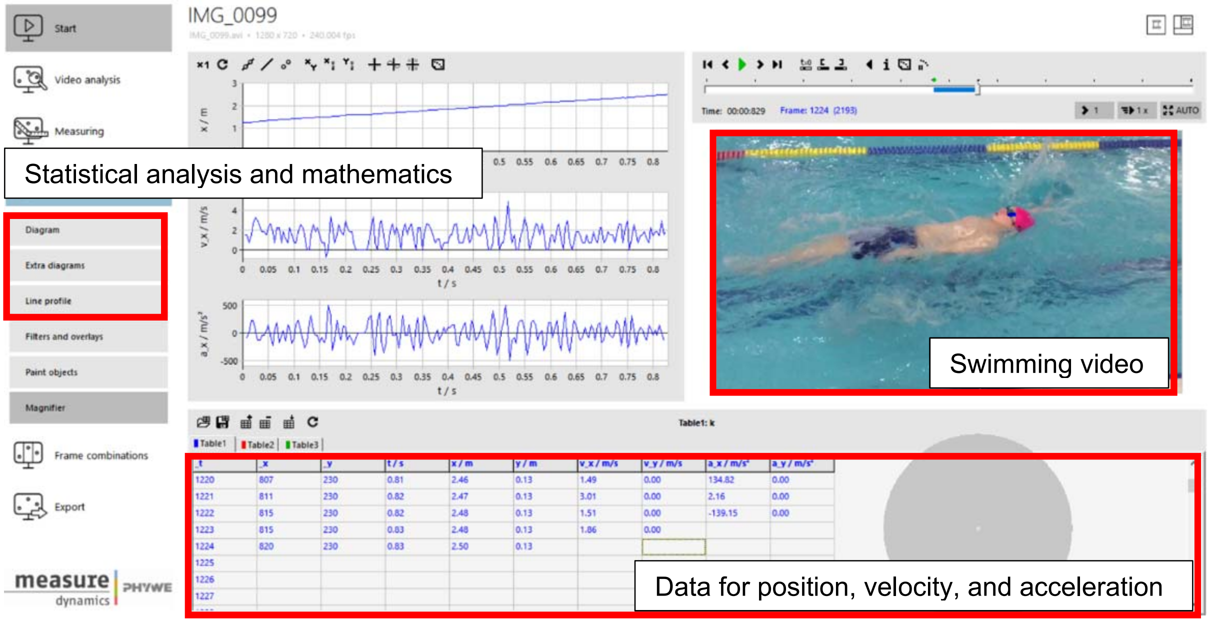
Given T, it is our objective to obtain ratio α for $\omega_2 = \omega_1 \alpha$ in which distance (the area under the curve $v(t)$) is maximized.

$$\frac{d}{d\omega_1} \int_0^T v(t)\, dt = 0 \tag{2}$$

Currently, assuming the ratio between A_1 and A_2 is experimentally obtained, this equation is a two-variable equation of $\omega_1$and α. For any input in the frequency value for either of the omegas, then we can obtain an α value in the single variable equation.

Through experiments we are able to obtain the ratio A_1: A_2, practically described as; the ratio of the strength of kick versus stroke.

1. *Procedure:* Video recordings were required to determine the amplitude ratio $A_1 : A_2$. The video analysis program 'Measure Dynamics' was utilized in this study. First, a body marker was placed on the cap of the swimmer. This program involves an automated program tracking the position of the body marker within the video on each frame relative to a designated origin, facilitating the calculation of velocity and acceleration based on the known time intervals between each frame. The velocity of the swimmer was recorded from the deck, above the swimmer in a fixed field of view. To measure the velocity contribution for an individual stroke we can implement the following experimental methods to make a fair approximation.
2. *Experimental Method:* We conducted experiments to find the velocity profiles for stroke and kick individually by:
   Recording streamline backstroke kick, perform backstroke kick while placing arms behind head, then kicking separately. Calculating velocity using Measure Dynamics program. Analysis of the velocity contributions of the arm and hand are performed separately.
3. *Data Analysis:* We used 'Measure Dynamics' program in Figure 1.



_t	_x	_y	t / s	x / m	y / m	v_x / m/s	v_y / m/s	a_x / m/s²	a_y / m/s²
1220	807	230	0.81	2.46	0.13	1.49	0.00	134.82	0.00
1221	811	230	0.82	2.47	0.13	3.01	0.00	2.16	0.00
1222	815	230	0.82	2.48	0.13	1.51	0.00	-139.15	0.00
1223	815	230	0.83	2.48	0.13	1.86	0.00		
1224	820	230	0.83	2.50	0.13				
1225									
1226									
1227									

Figure1: Analysis of the position, velocity, and acceleration as a function of time by the Measure Dynamics program

We tracked a single body marker on the swimmer's cap and automatically track the movement of that point frame-by-frame (Figure 2). The 'Measure Dynamics′ program calculated the change in pixels by unit frame. Then, with the reference of the swimming lane behind (1 m for each blue, yellow section), the program converted velocity values in m/s. An IPHONE 6 was used for recordings and slow motion application (240 fps) was used.

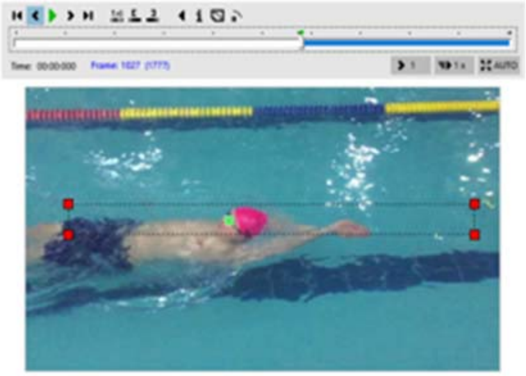


Figure 2: Sample picture for a trial on backstroke without kick. There is a green marker on the swimmer's cap for tracking.

RESULTS: In the velocity data, there are peaks and troughs. The average value of the peaks is expected to be A_1 and A_2, respectively. The calculated average value for the peaks over the five trials was determined (Table 1).

Table 1: Maximum velocity over five trials.

Trial	Arms (m/s)	Legs (m/s)
1	1.989	1.012
2	2.118	1.019
3	2.047	0.989
4	1.994	0.978
5	2.309	0.895
Mean	2.091	0.978

Stroke velocity was 2.091 m.s (peak velocity), kick velocity was 0.978 m.s (peak velocity), and their amplitude ratio is 2.091:0.978=2.25:1. In other words, for backstroke, the arm is 2.25 times more influential to velocity. This is $A_1 : A_2$ .We then compute the ratio $\omega_1 : \omega_2$ that maximizes the integral of the function $|A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t + \varphi)|$ for a specified duration T, in other words, the distance travelled by the velocity gained by stroke and kick. The mathematical task here involves finding the optimal $\omega_1 : \omega_2$ (since we know $A_1 : A_2$) that will maximize the area under the curve from 0 to the time T. This involves solving Equation (2), where $A_1 = 2.25$, $A_2 = 1$, and $\omega_2 = \omega_1 \alpha$. Currently, this function is a two-variable function of $\omega_1$and α. By inputting an individual's frequency value for either of the omegas, we can obtain an α value in the single variable function and thereby obtain the maxima. Therefore, for a swimmer who knows their average strength of stroke to kick ratio to be approximately 2.25:1, they must stroke and kick at a ratio of α, which can be obtained by the procedures and theories explained above.

DISCUSSION: The presented approach relied on a series of assumptions which can be improved over time. First, it is hard to assume the propulsion of a swimmer simply as a sine function. Although the propulsion of continuous strokes may be periodic, alternative forms of propulsion must be considered to increase the accuracy of this method. Another aspect to be considered is the possible interaction between kick and stroke, which effect was assumed to be insignificant in this paper. Finally, it was not considered if this ratio is energy efficient. It is important that these frequencies be sustainable to maximize the efficiency of the swimmer.

CONCLUSION: Like freestyle swimmers, it is important that backstroke swimmers have a designated kick to stroke ratio for guidance. This paper presents a general method to coaches and future researchers. As this experiment was conducted upon a few amateur swimmers, it is hard to generalize the 2.25:1 amplitude ratio, but the methodology itself is valuable. Therefore, with the knowledge of the relative strength of arm and leg, coaches and prospective swimmers can find a function for finding the optimal ratio of stroke to kick for themselves.

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ACKNOWLEDGEMENTS: Sebin Wang and Sehyun Wang of Hongchun High School have helped as demonstrators within this study.

EDITORS' NOTE: This paper is a special acceptance for a school student, who had the initiative to submit a paper without telling his father, who is a Professor of Robotics! In the international spirit of encouraging students to learn biomechanics, this paper has been accepted at the conference. We do have to note that the methods in the paper are unfortunately flawed as the camera position was not perpendicular to the swimmer for the calculation of velocity, and the calibration was the lane rope well behind the swimmer. The maths in the paper is correct, however, the input data contain errors.